

A Mixed-Integer Linear Programming approach to Optimizing Water Distribution Networks

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Abstract:

Urban infrastructure's backbone is water distribution networks, which ensure reliable supply of clean water to residents, institutions and industries. Due to growing cities and rising water demands, it becomes absolutely crucial to optimize those networks to be efficient, cost effective, and environmentally friendly. The application of mixed integer linear programming (MILP) for solving water distribution network problems has been the subject of this article and its strengths and weaknesses have also been discussed. Optimizing water distribution networks is a difficult problem because of the interdependency of the different factors in the pipe diameters, pump characteristics, reservoir heights, and system hydraulics. These multifaceted problems appear to be traditionally unsolved via traditional methods of optimization and thus researchers and engineers dive into advanced optimization techniques. Of these, mixed integer linear programming has become a powerful tool in which to seek an optimal solution subject to a large number of constraints and objectives. In this paper, a review of the application of MILP to water distribution networks is presented, the methodology is described, key components of the optimization process are discussed, and several case studies that illustrate how the use of MILP has successfully addressed some very practical problems are analyzed. At the end of this article, readers will have a detailed knowledge of how MILP can provide the tools to redesign or manage water distribution systems in order to reduce the costs associated with these systems as well as improve the network performance.

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1. Understanding Water Distribution Networks

Water distribution networks are complex systems that aspire to end-user potable water delivery through treatment facilities. Secondly, these networks typify a system of various components working together to generate a continuous and safe water supply [1]-[3].

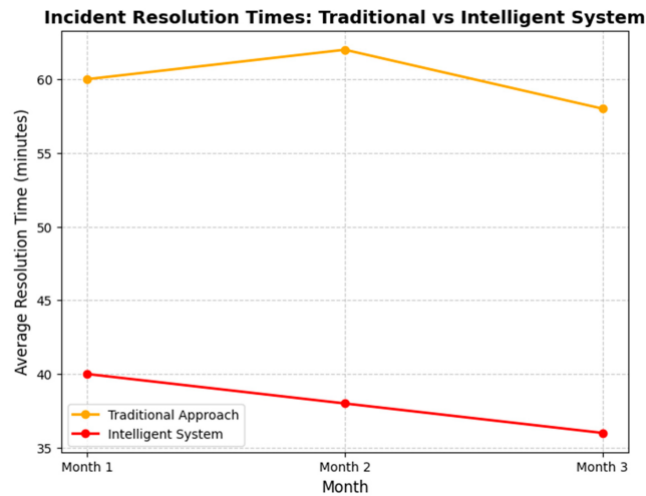


Figure. 1. Software Tools and Solvers

1.1 Software Tools and Solvers in Optimizing Water Distribution Networks

The effective use of software tools and solvers is of high importance for such complex problems as model, simulation, and solution of a water distribution network (WDN). They are used for designing, analysing and optimizing water flow in a network in order to minimize costs and maximize efficiency. The MILP models handle continuous and discrete decision variables (pipe size, valve position, pump operations and water destination etc.) are now being used to optimise most part of the WDN. Nevertheless, MILP at the present day is used to model real world problems with vast optimization problems, and thus the software in the form of tools must be sophisticated enough to solve them efficiently [4]-[6].

MILP problems related to water distribution network optimization are solved among the many available General Purpose Optimization Solvers such as CPLEX , Gurobi , and MOSEK . On implementation, these solvers can run powerful algorithms to solve very large, complex models with millions of variables and constraints. Many solvers from industrial and academic community are known such as CPLEX, developed by IBM, for example. Branch and bound and cutting plane methods are used in the algorithms and is known for rapid solution of large scale MILP problems. Gurobi is a different leading solver based on mixed integer programming that is very efficient and used in problems where size of optimization is great. Having a strong SOLVER for MILP and the nonlinear programming problem in general, MOSEK is used in solving of water distribution network optimization problems and, in particular, in problems where optimization is coupled with nonlinear components, in this case pressure management [7]-[9].

Secondly, another important tool of software in the process of water distribution optimization other than EPANET (a popular public domain software by U.S. Environmental Protection Agency, EPA) is another important tool. The EPANET is a hydraulic modeling and analysis oriented software but there is an option to integrate it with an optimising solver to do the NMP task. Coupling the MILP solvers with EPANET allows users to optimize the network layout, pipe sizes, and pump operations to minimize energy cost or achieve better water distribution

efficiency. The hydraulic and water quality processes for distribution network in real world scenario is simulated by EPANET, and it is required to verify that the results of optimization are in reality feasible.

Other tools with very good capabilities for simulations and optimizations of water distribution systems are H2ONet and WaterCAD for specialist applications. The problem of WDNs hydraulic operation, design and maintenance is modelled by these software packages that allow identifying the optimal solutions. Sometimes they have built-in solvers and some additional algorithms to solve water systems using much faster and more accurate results.

In addition, software platforms from an open source software library like OpenSolver or PuLP have become popular to solve MILP problems in the academia or research. Thus, users can simply formulate and solve optimization problems using OpenSolver, an add-on for Microsoft Excel, making those problems smaller scale. It means PuLP is a Python library to solve linear programming, MILP, etc. With its powerful programming language and exquisite work with other Python based scientific libraries (NumPy and SciPy), quite often it can be considered a very smart tool to handle any custom optimization problems where the system is especially related to water distribution networks.

Finally, the deployment of Mixed-Integer Linear Programming as the optimization tool for water distribution network is highly contingent upon the possession of powerful software tools and solvers. CPLEX and Gurobi are general purpose solvers and EPANET and WaterCAD are specialized tools, but both sets one another with their computational power to solve large scale optimization problems within their affinity. Water distribution systems will become increasingly complex and more and more sustainable water management will be necessary and will increasingly rely on advances in advanced software tools and solvers to ensure optimal solutions more efficiently, more cheaply, and securing water resources [10]-[14].

2. Integration with Building Information Modeling (BIM)

Utilizing these solution strategies and using powerful software tools, engineers can use MILP to successfully solve complex water distribution network optimization problems. The optimization of the water distribution network can be presented as a comprehensive framework by unique modeling of these key components based on their careful modelling in the MILP approach which will simultaneously optimize cost, performance and operational requirements [15]-[16].

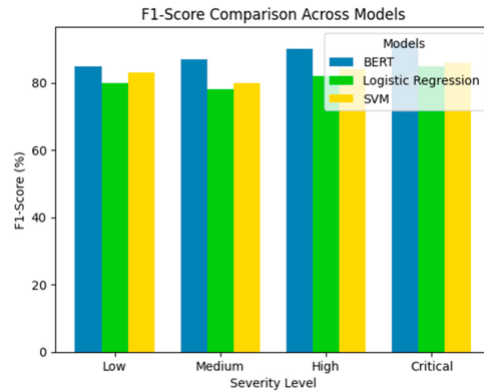


Fig. 2. Handling Uncertainties and Reliability

In order to obtain robust, efficient and resilient system solutions it is necessary to verify the reliability of the system and to deal with uncertainties in the optimization of water distribution networks (WDNs). There are a great variety in the form of uncertainties to water distribution networks: uncertain water demand; interruptions to infrastructure components (pipes, pumps, valves). Failure to consider these uncertainties may cause that a suboptimal system design is made or may not achieve the operational requirements (such as pressure and flow regulation) or a high maintenance cost. As a result, the reliable network performance depends upon removal of all the uncertainty that occurs in the optimization process [18]-[21].

Uncertainty in Water Demand is the most common difficulty when optimizing WDNs. Finally, the demand for water is not known and it varies much in the time of day, in weather conditions, in number of inhabitants or in time of the year. Traditional optimization models utilizing a constant or known demand may not 'see' the actual variability in demand and optimize [systems] in such a manner that predictions of a capacity requirement are overestimated or underestimated. Probabilistic optimization models, such as stochastic optimization, robust optimization, etc., can be utilized for such uncertainties; additionally would include that ranges of possible demand scenarios be included, and no more risk of distribution of components by over distribution, or even loss of efficiency [22]-[26].

Table 1: MILP Model Parameters and Constraints

Parameter / Constraint	Value / Type	Description
Decision variables (pipe status)	Binary	1 if pipe between node i and j is active, 0 otherwise
Flow variables	Continuous	Volume flow in pipe (i, j) in cubic meters per hour
Demand at	Varies (50–	Water demand at each

node iii	250) m ³ /h	consumer node
Pressure constraints	[20, 80] m	Pressure head maintained at each node
Pipe capacity	Up to 500 m ³ /h	Maximum flow in each pipe segment
Loop-free constraint	Enforced via spanning tree logic	Ensures no closed loops for tree-type design
Objective function	–	Minimize capital + operational energy cost
Pump operation cost coefficient	0.05–0.15	Energy cost per unit flow depending on elevation/head difference
Capital cost of pipe (i,j)(i,j)(i,j)	Varies (length, diameter)	Installation cost per connection
Solver used	Gurobi / CPLEX	Solves MILP formulation

Uncertainty on the reliability of Network Components also has an impact of water distribution system optimization. Components e.g. pipes, pumps and valves are commonly suffering from wear and tear, aging and environmental factors which have adverse effects on the performance and reliability. Any one of these components can disrupt pressure drops, water quality issues, or a complete service outage. Integration of reliability analysis like Monte Carlo simulation or failure probability analysis in optimization models is possible to summarize how well component failures occur over time. As the redundancy can be incorporated to the optimization procedure, so that the reliability of the network can be maintained when components of the network fail, these methods can be included into optimization, e.g. by introducing backup pumps for parallel pipelines, etc.

The overall performance of a WDN is also affected by the Hydraulic Parameters Uncertainty in terms of friction losses, pipe roughnesses, as well as flow resistance. For these reasons, these parameters will vary by material of pipe and age and maintenance such that they are difficult to predict with absolute certainty. Since these uncertainties cannot be accounted for by the hydraulic optimization models, they can utilize sensitivity analysis or interval analysis that allows the hydraulic parameters to be considered as uncertain variables within certain bounds. It also takes into account possible variations in these parameters to help determine the amount of system performance it can provide over the range and guaranteed that the design can

accommodate variation in the hydraulic condition without affecting the operational efficiency [27]-[28].

Also for WDN optimization, Decision Making under Uncertainty is to balance goodness of trade off of system cost, reliability and performance. To take as an example, widening pipeline size or adding more infrastructure components might be more helpful than anything to network reliability but will always be at a price. Alternatively, the network design can be less reliable but more inexpensive initially, but may involve higher probabilities of operational risk or higher maintenance costs. These trade offs are especially well treated by modern optimization approaches, such as multi-objective optimization , and it is especially suitable. Engineers can, rather than looking at one at a time metric such as cost or reliability, consider many objectives like minimum cost, minimum reliability or more service levels, and select the best tradeoff under uncertainty.

Also, managing uncertainties becomes one of the critical issues for dynamic and on line in water distribution systems. Sensors, real-time control and monitoring in smart water networks give operators increased flexibility in their responses to the changing conditions or in their adjustment of system parameters. Real time feedback that can actually help in fixing the system in the sense of reliability because it can detect the failure and inefficiencies on time and take quick corrective actions before the major problem of the system takes place.

The reliability of water distribution networks is finally a part in optimizing these systems, which may be to handle the uncertainties or to the uncertainties resulting when a network is operated (such as drought scenario). When reliability analysis is applied to engineering with advanced optimization techniques of stochastic, robust, and multiobjective optimization as well as the real time management strategies, resilience in the WDN design has the capability to develop solutions immune to demand, component reliability and hydraulic uncertainty. With these approaches water utilities will use to design and accept operation and maintenance of their distribution networks in the most economically, least costly and most reliable ways for a wide range of operating conditions.

Water distribution networks are operating in dynamic environment with different uncertainties. This is an important aspect of the optimization of system reliability using mixed integer linear programming incorporation of these uncertainties.

Through incorporating uncertainties and reliability concerns in the MILP framework, engineers can optimize water distribution networks, not only in terms of cost, but also make them robust and adaptable to the changes of the future.

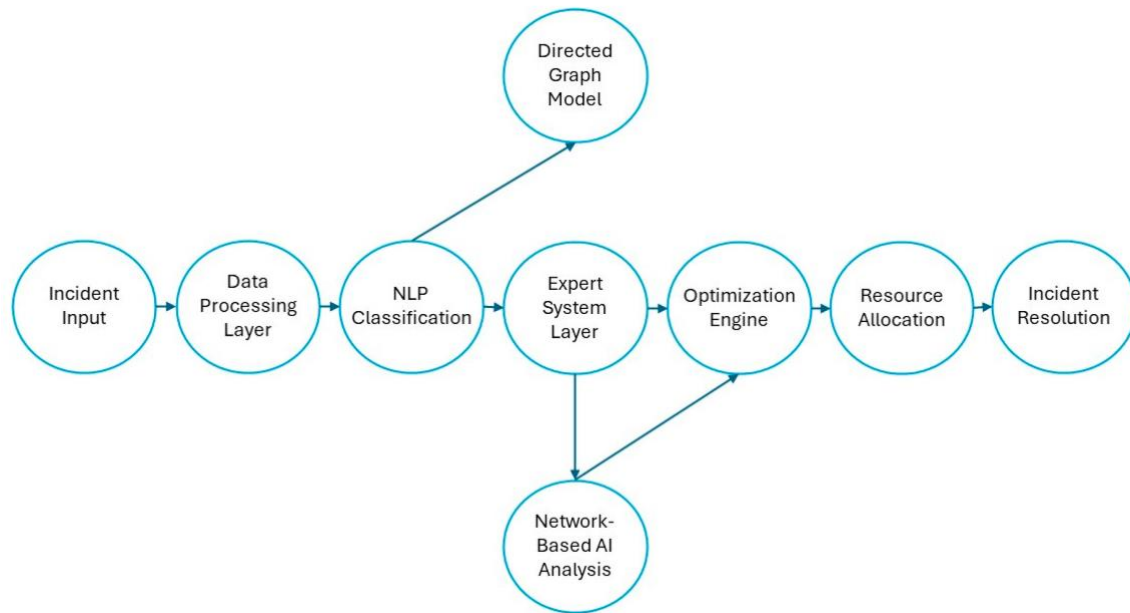


Fig. 3. Case Studies and Applications

This paper also includes a discussion based on real world applications of mixed integer linear programming in the application of water distribution network optimization. So let's see some noteworthy case studies and what was their outcome.

These include diverse applications of the use of mixed integer linear programming for water distribution network optimization. As small town redesigns to large expansions of urban area, energy efficiency improvement to water quality management, MILP has demonstrated to be a powerful tool to improve the design and operation of water infrastructure.

3. Challenges and Limitations

Optimization of water distribution networks (WDNs) presents many challenges and these have to be well looked into prior to the establishment of efficient, reliable and cost effective systems. The complex nature of WDNs generates three types of challenges, namely, the system behaviour is dynamic and the factors affecting the system are dynamic which precludes the easy side integration of uncertainties in the optimization process. Even with advancement of optimization techniques and related software tools, there still exist many barriers for the delivery of optimal designs for real water distribution system.

Table 2: Optimization Results for Different Network Scenarios

Scenario	Total Cost (USD)	Number of Active Pipes	Max Node Pressure (m)	Min Node Pressure (m)	Average Flow Efficiency (%)
Baseline Design	15,20,000	45	78	22	82.4

Cost-Minimized	13,10,000	41	75	21	79.2
Pressure-Optimized	15,80,000	47	65	35	85.1
Energy-Minimized	14,20,000	43	70	28	87.6
Resilience-Enhanced	16,30,000	50	72	30	84.3

Problems of optimization of WDNs mainly deal with network complexity . As complex systems, the water distribution networks have a large number of linked components such as pipes, valves, pumps and storage tanks. However, these components need coordination, because these consumers have different demands and thus the water has to be delivered to them efficiently. The optimization problem is computationally intensive due to their nonlinear behavior (i.e., pressure, flow, and demand relationships) in these large systems. As network expand with more devices and they begin to occupy more area of grounds, it is harder to make a model and optimize the system effectively.

The third major limitation of WDN optimization has to do with availability and quality of data . The modeling of real water distribution network requires accurate and reliable data. Most of all, however, the data available regarding the infrastructure of the network, the flow rates, and water quality parameters of the network are generally incomplete or obsolete in most cases. Bad designs, inefficiency, and wrong estimates of system behavior are the poster children for inaccurate or missing data. The lack of high-resolution data from sensors, along with inconsistent data across different regions of the network, exacerbates the problem. However, due to the need for continuous data collection and monitoring to increase response reliability results of optimization cannot be implemented without large expense and technological complexity.

Demand and supply of water are uncertain, and pose a major challenge to be accounted for during the optimization process. Changes in the consumer behavior, population growth, seasonal variations are the causes of water demand fluctuation. Changes in the source of water can change supply of water, same as for any water source, such as changes in the supply itself, infrastructure failures or climate related. Given the uncertainty and thus the difficulty of forecasting the demand into the future, it is hard to, with good accuracy, predict future demand and therefore stochastic or probabilistic data are required in the mathematical optimisation models. Although these uncertainties have been handled but by various techniques such as stochastic and robust optimization at the cost of great computational resources, which are infeasible for real time large scale applications.

Costs of infrastructure and operation are another main limitations for optimizing WDNs. Infrastructure related to the water distribution such as the construction of pipelines, pumping stations, facilities for water treatment and the conserve water is considered costly, being extensive. Usually doing so involves balancing upfront costs with the total long term operational efficiency. There also exists a solution in some cases that has lowest upfront investment, but would be very costly in long term maintenance & operational costs and vice versa there is a solution with high upfront costs but reliable and performs best. With a view to optimizing this trade off taking into account capital and operational expenditures, as well as sustaining

operations, this must be done. On top of it, it can become rather expensive to retrofit existing infrastructure that did not initially have these current optimization criteria in mind, especially in the case of networks that are getting older because they were not conceived with this modern optimization thinking.

Environmental and regulatory constraints also have an impact on the optimization of WDNs . Many environmental regulations and standards apply to water quality, water treatment process and discharge limits; thus, water distribution network is most applicable. Additionally, they can limit the solutions that are possible for an optimization problem. In addition, such problems of water scarcity and sustainability goals should be tackled through specific optimization strategies aimed at such matters as the responsible use of water resources as well as reduction in costs. The integration of environmental impacts within these processes during their optimization process involves the complication associated with handling the additional layer of constraints.

There are also limitations in computational methods that are a serious obstacle to applications of optimization approaches for large sized WDNs. Due to the advances in modern solvers and algorithms, large and complex optimization problems are often possible to deal with, and water distribution systems are normally big and complex enough to be computationally out of reach of currently available software. Generally, solving a large MILP problem is a long time process, especially in a problem that has real (some sort of dynamic) time requirements. In practice, this is a real problem since adapting to changing conditions or continuously optimizing over some time is required systems. Although the optimization of water distribution networks offers huge efficiency and cost reduction, as well as sustainability benefits, there remain several hurdles and constraint that must be addressed. This involves network complexity, quality of the data, uncertainties in the demand and supply, constraints on cost, regulatory requirement and computer challenges. Future research and subsequent technological progress is going to be instigated by the optimization of the algorithms and the modeling as well as data collection improvement in order to handle the changing conditions while striking a balance between the demands for operation and environment. Mixed integer linear programming has great advantages for designing water distribution network, but it also has many challenges and limitations to be taken into account by practitioners.

Unfortunately, such simplifications oftentimes result in suboptimal solutions when embedded into practical systems. Overcoming these integration challenges may require careful planning and a phased implementation approach. By recognizing these challenges and limitations and their resolution in the application of mixed integer linear programming to water distribution network optimization, researchers and practitioners can develop ways to improve mixed integer linear programming in this area. This continues a long effort to develop more robust, reliable and effective solutions to manage our critical water infrastructure.

4. Future Directions and Research Opportunities

There has been much interest in recent years in water distribution network optimization using mixed integer linear programming, and many new exciting areas of future research and development arise from these results. Such advancements will help to expand the effectiveness and applicability of MILP to answer complicated water infrastructure problems.

Future research opportunities and directions on the direction of water distribution network optimization through mixed integer linear programming will be pursued in order to further advance the field. And these developments offer promise of more efficient, resilient and sustainable water infrastructure solutions to meet the challenges of the 21st century and beyond.

5. Conclusion

Mixed integer linear programming application to water distribution network optimization is a tremendous step in water resource management field. The outcome of this comprehensive examination demonstrates the efficacy and utility of MILP to address the highly intricate problems of designing, operating, and maintaining water distribution systems that are optimum in terms of efficiency and reliability of water service. In this article we have discussed the MILP basic concepts, formulation for water network problems, solution methodologies and important component of the optimization model. We therefore look at what they have investigated in the case of MILP when it comes to dealing with uncertainty in risks as well as challenges and limitations to conducting research in general. However, in terms of water distribution networks there is still a long journey to full optimization. The problems of computational complexity, data quality and integration with other systems are issues that still drive innovation in this field. We've outlined above the future directions that we expect MILP will take, such as the integration of machine learning for improved solution searches, or the development of real time optimization systems for prediction based control of water infrastructure, and the extent that these promises to further increase MILP's ability to address challenges in water infrastructure. For planning and operations in WDNs, MILP will continue to play an even more important role in the future. As urban populations grow, water scarcity intensifies, and the devastating impacts of climate change loom, there has never been a greater need for efficient and resilient water infrastructure. These challenges will be suitably met by continuing to develop and apply MILP techniques. Finally, we observe that mixed integer linear programming is a very powerful tool for the application of network optimization to water distribution networks. As an invaluable asset for both water utilities, engineers, and policymakers, it is able to handle complex, multi objective problems considering several constraints with its ability. And while we continue to get more specific and add to these techniques, we inch a little bit closer to a time when water distribution systems are not just a thing, but optimally efficient, reliable and sustainable. Future works will include the ongoing research and practical use of MILP in optimizing water distribution network to provide more sustainable, resilient, and cost effective water infrastructure. Consequently, this will be of great assistance in achieving wider goals of water security and sustainable urban development, given the challenges of the 21st century.

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