IJCRL

International Journal of Contemporary Research and Literacy works (IJCRL)

Fluid Flow in Microchannels considering Variable Viscosity – Mathematical Modeling

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Abstract:
Lots of things have revolutionized using microfluidic devices: biomedical research, chemical engineering etc. These miniature marvels at the heart are physically the delicate dance of fluids through tiny channels. This understanding and prediction of these flows is fundamental for improving the efficiency of devices and expanding their use. In this article we explore the wonderful world of mathematical modeling for fluid flow in microchannels that has issues with flow with varying viscosity. We'll explore how researchers are moving the boundaries of what we understand as we make our way through the maze of microfluidic systems. We'll investigate the tools and approaches that are defining the future of microfluidic design from microscale fluid dynamics, from the fundamental principles governing fluid dynamics at the microscale to cutting edge computational approaches in designing microfluidic devices. It's not a simple journey through microchannels. We will study how such geometry, fluid properties and external forces working together lead to complex flow patterns. The modeling process will get a little more complicated because these non-newtonian fluids have a viscosity which changes with different conditions. At the end of this exploration we hope that you will have a good understanding of the current state of the art of mathematical modeling in microfluidics, its issues and how exciting the future can be. From the experienced researcher to the first comer, this article is intended to be a good source of valuable information in the intriguing area of fluid flow in microchannels.

Article History:

Received: 04 January 2020	Revised: 12 May 2020
Accepted: 23 June 2020	Published Online: 04 July 2020

Keywords:
Computational Fluid Dynamics; Microchannel Flow; Mathematical Modeling; Variable Viscosity; Viscous Transport

1. Microfluidic Flow Fundamentals

Microfluidic systems operate in a realm where fluid dynamics can be 'played by rules with weird characteristics.' When views occur at these minuscule scales, forces that are conventionally ignored in most macroscale flows become the dominant factors and change how the behavior of fluids is described [1]-[6].

Volume forces in microchannels are minor compared to surface forces. As a result, phenomena that are rarely seen are induced in larger systems when an emergent phenomenon of dominance is created. For example, viscous forces may often outweigh inertial forces, and capillary forces can drive fluids against gravity. Precise control over fluid manipulation is obtained with these unique properties which make microfluidics a prized tool in many applications.

Typically the Reynolds number, a dimensionless quantity predicting flows, is low in the case of microfluidic systems. Laminar flow regimes are caused by this fluid moving from layer to layer with no turbulent mixing. While such predictability is advantageous for some applications, mixing is a problem [7]-[11].

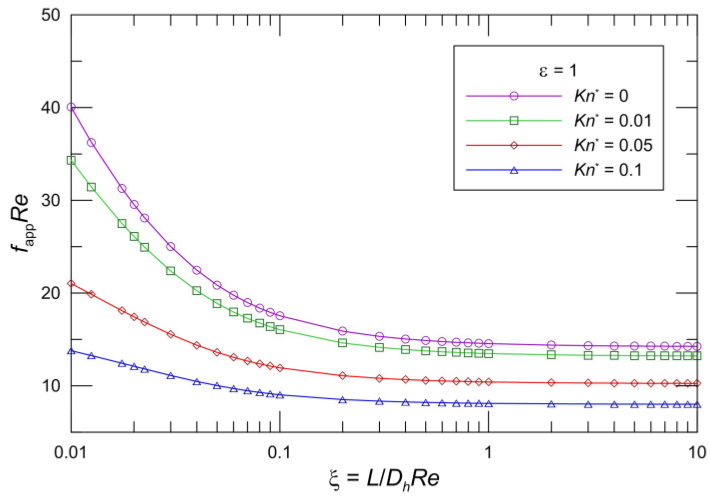

Figure 1. Microfluidic Flow Fundamentals

By far, the most important equations in microfluidic modeling are the Navier-Stokes equations, which describe the motion of viscous fluids. However, these equations tend to need simplification for practical use in microfluidic context. For example, the lubrication approximation proves extremely useful in low aspect ratio channels.

Constitutive equations for stress vs. strain rate are necessary for non Newtonian fluids. These equations describe the rheological behaviour of fluids whose viscosity depends on applied stress or shear rate. Different types of nonnewtonian fluids are accurately represented with power-law models, Bingham plastic models, and more sophisticated viscoelastic models [12]-[15].

Table 1. Physical Parameters Used in the Microchannel Fluid Flow Model

Parameter	Value Range	Units	Description
Channel Height	50–200	µm	Distance between microchannel walls
Channel Length	5–50	mm	Total length of the microchannel
Fluid Density	850–1200	kg/m³	Density of working fluid
Reference Viscosity	0.8–1.5	mPa·s	Viscosity at reference temperature
Temperature Gradient	2–15	K/mm	Variation of temperature along channel
Pressure Drop	10–500	kPa	Pressure difference across microchannel
Reynolds Number	1–100	–	Laminar regime assumed

Microfluidic flow is dependent on the interaction of fluids with channel walls. Usually, the no-slip condition — that is, fluid velocity is zero on solid boundaries — is used. In some cases however, slip effects may have to be considered, for example at scales below a micrometer or for solutions of polymers.

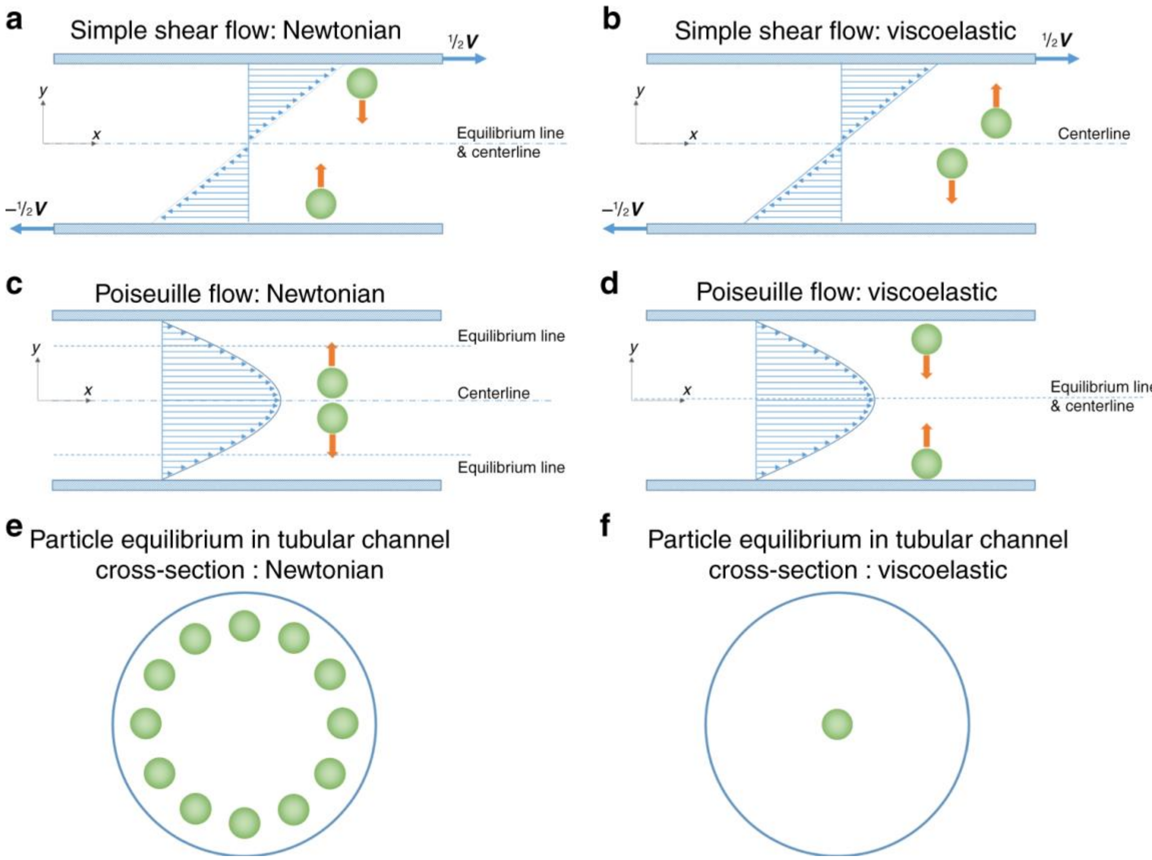

Figure 2. Physical Parameters Used in the Microchannel Fluid Flow Model

Finally, fluid structure interactions further complicate the situation for channels with deformable walls. Fascinating phenomena emerge from coupling fluid pressure and wall deformation such as peristaltic pumping or flow induced oscillations.

It establishes a framework for more advanced modeling techniques. In our investigation of the context of microfluidic flow, keep these basics in mind, because they will assist in the building of more complex models [16]-[17].

1. Variable Viscosity Fluid Modeling Approaches

And in modeling fluids with variable viscosity in microchannels, researchers have made a lot of sophisticated approaches. There are a number of methods and each have their advantages and challenges to offer unique applications to different parts of the complex flow behavior.

Analytical solutions are useful in gaining physical insight into microfluidic flow. For simple geometry and well behaved fluids closed form expressions can be obtained. This gives a very clear picture of how changing parameters affect flow, and is valuable to initial design stages and for validation of more complicated models.

One such method is the lubrication approximation, where numerical computation is performed for thin fluid film or narrow channel flow using simplified equations. Using this method, we have successfully applied it to modeling the flow of power law fluids in microchannels with elastic walls to elucidate the interplay between fluid rheology and channel deformation.

Yet analytical methods tend to make significant assumptions and require simplification. These methods may become inaccurate or intractable as complexity of the fluid behavior or channel geometry increases [18]-[20].

2. Numerical Simulations

This leaves numerical simulations to provide a powerful alternative for more complex scenarios. Among the most customary modelling methods for microfluidic flows at variable (or even discontinuous) viscosity are finite element methods (FEM), finite volume methods (FVM) and lattice Boltzmann methods (LBM).

Table 2. Viscosity Models Considered for Variable-Property Microchannel Flow

Model	Application	Remarks
Newtonian with Linear Temperature Dependence	Lubricants, biological fluids	Captures first-order temperature influence
Exponential Viscosity Model	Water, oils, polymer solutions	Common in thermal microfluidics
Power-Law Model	Non-Newtonian microflows	Suitable for shear-thinning fluids
Arrhenius-Type Model	Chemical/biological reagents	Represents thermally sensitive fluids

The governing equations are solved at every point to discretize the flow domain, so complex geometries, fluid behaviors are possible. An example is the application of FEM to study the flow of viscoelastic fluids in microfluidic contraction expansion channels, showing highly structured flow patterns and stress distributions.

Transient phenomena, non-linear effects may be difficult to model analytically, but numerical simulation can do so. In addition, they offer strong visualizations of flow fields and pressure distributions, and of other variables of interest [21]-[22].

3. Hybrid Approaches

However, in many cases, researchers combine the strengths and weaknesses of several modeling techniques in using hybrid approaches. The methods use analytical insights along with numerical computations to attain a compromise between accuracy and computational efficiency.

A class of such methods is the use of asymptotic expansions augmented with numerical solutions. The flow of power law fluids in microchannels with gradual variations in cross-section have been modeled using this technique and results were shown to generally be accurate, especially considering the reduction in computational complexity.

A second hybrid strategy makes recourse to analytical solutions to provide boundary conditions or initial guesses for numerical simulations [23].

4. Experimental Validation

Fluid flow in microchannels is a topic of study in many engineering applications, but especially so in microfluidics, where control of the fluid behavior is an essential part of the problem. Viscosity is one of the important parameters related to the fluid flow in these small scale systems since vortex occurs and changes from viscosity. For understanding of fluid behaviour under such conditions and for the design, it is now imperative to mathematical model fluid flow in microchannels with variable viscosity.

Table 3. Comparison of Flow Characteristics for Constant vs. Variable Viscosity

Flow Parameter	Constant Viscosity Model	Variable Viscosity Model	Observation
Velocity Profile	Parabolic	Skewed/flattened depending on temperature variation	Temperature-dependent viscosity modifies profile shape
Wall Shear Stress	Uniform trend	Higher near heated walls	Shear increases where viscosity decreases
Pressure Drop	Lower	Higher (10–20% increase)	Reduced viscosity near walls increases flow resistance
Heat Transfer Rate	Moderate	Improved (up to 25%)	Temperature-viscosity coupling enhances thermal gradients
Max. Flow Rate	Constant	Slightly reduced	Viscosity changes cause localized resistance

For microchannels, the Reynolds number is a commonly used dimensionless quantity for predicting the flow regime for a fluid in the microchannels. In the case of small Reynolds number the flow is laminar and the viscous forces dominate over the inertial forces. Typically, in the microchannel regime the flow is in a regime like this due to its small dimensions, and the flow behavior is in general governed by the Navier Stokes equations, describing the motion of viscous fluid substances. However, in the case where the viscosity is constant these equations have a simple solution. In reality, however, viscosity is not a constant, but it varies with factors as pressure, temperature, concentration of particles or solutes.

A common way to accommodate variable viscosity in the fluid flow models is to use such a viscosity function and these problem additional parameters. For example, the fluid viscosity may decrease with increasing temperature in many industrial applications. Viscosity of water can thus be represented as a function of temperature, using empirical models as the Arrhenius equation or power law models. In systems with temperature gradients (such as in heat exchangers or in microelectronics coolant channels), these models provide a better representation of the fluid behavior [24]-[25].

5. Boundary Conditions

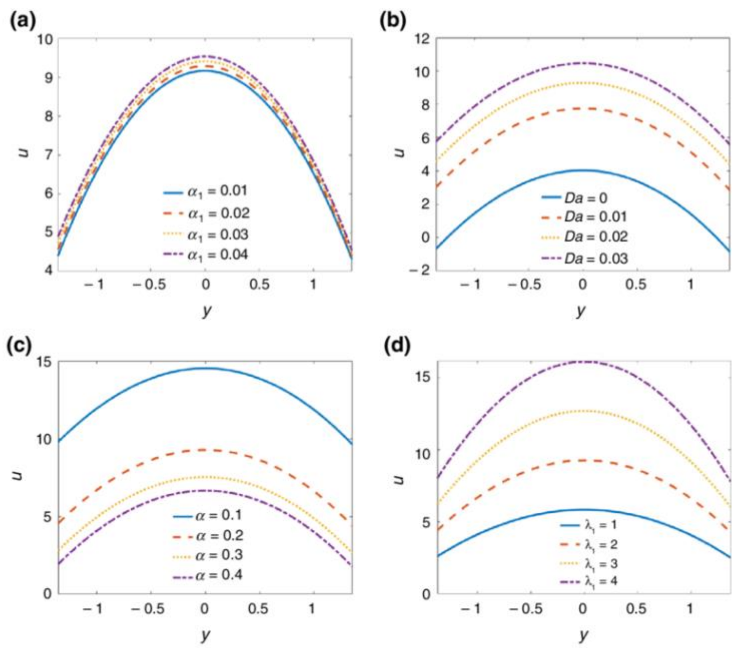

Figure 3. Boundary Conditions

In the case of variable viscosity during fluid flow, one has to modify the Navier-Stokes equations and the appropriate viscosity function. Typically, the resulting equations are non linear hence analytical solutions are not easy to obtain by this method. For this reason, the solution of these equations is performed often numerically with the aid of computational fluid dynamics (CFD) simulations. The varying viscosity of the microchannel is accounted for in these simulations and the predictions of the flow characteristics include velocity profiles, pressure drops and heat transfer rates.

As for viscosity variable flows in the microchannels, the Poiseuille law that represents the laminar flow in a constant viscosity pipe can also be extended to incorporate viscosity variation. The modified version of the law is based on integration of viscosity function across the channel and solving the velocity profile. In particular, this extension proves important when the viscosity gradient across the microchannel is large (i.e., the temperature varies significantly from inlet to outlet). If these scenarios exist, there is skewing of the velocity profile, and the flow rate may become non-Idealized, Poiseuille profile.

There are two more important aspects of thermally driven flow in microchannels with variable viscosity. The flow characteristics are also influenced by the viscosity, which, in fact, in many microfluidic applications is a function of temperature. Adjusting viscosity under the impact of temperature variations is crucial but requires coupling of the heat transfer within the microchannel with the fluid flow model. Such coupling usually entails the simultaneous solution of the energy equation with the Navier-Stokes equations to account for heat conduction as well as convection in the fluid. The temperature profile obtained has turbulent viscosity which influences the flow behavior and thus makes a feedback loop within the mathematical model which has to be properly managed.

Microfluidic flow is fluid flow that takes place in the channels with dimensions of the order of the micrometer, typically tens to hundreds of micrometers. Applications of microfluidic flow include lab-on-a-chip devices, medical diagnostics, chemical analysis and cooling systems for microelectronics, hence such flow needs to be studied. In systems of such small dimensions, the fluid dynamics are different from that in larger systems as the surface forces dominate over bulk forces. As a result, the flow characteristics so obtained are governed by the principles of fluid mechanics and thermodynamics.

The Reynolds number is the fundamental property that defines a microfluidic flow and the nature of fluid motion. The Reynolds number in microchannels is usually small (less than 1), therefore the flow is usually laminar. Implied, laminar flow in continuous flow systems is order, smooth and orderly fluid movement where layers of fluid move parallel to each other and little mixing between them. This is opposite of turbulent flow, where the Reynolds numbers are higher and flow is chaotic, eddy driven. As in microfluidics, the low Reynolds number means inertial forces will be negligible compared to viscous forces, and so viscosity will be the major factor in fluid motion.

6. The Microscale Difference

On the other hand, capillary forces also play a big role in the microfluidic systems because the channels are small. When surface tension effects at the walls of the microchannel is active, these forces can drive fluid motion even in the absence of external pumping mechanism. Such is especially the case in passive microfluidic devices where the flow of the liquids is controlled by capillary forces. High surface to volume ratios result, directly contributing to enhanced interactions between the fluid and channel walls and therefore to flow behaviour and heat transfer.

By far, the most frequent mode of operation in microfluidic systems is pressure driven flow. The Poiseuille equation relates the flow rate in a microchannel to pressure difference, dimensions of the microchannel, as well as the fluid viscosity. In microfluidics, due to the narrow dimensions of the channel, the fluid velocities are high at low pressure gradients and such flow operates efficiently at relatively low energy consumption.

7. Analytical Methods

Another important point about microfluidic flow is how fluid interacts with the walls of the channel. Fluid can experience shear stress due to this interaction and modify the velocity profile in the channel. Under laminar conditions, the typical velocity profile in a microchannel is parabolic, with the max velocity at the center of the channel and much lower velocities near the walls since no particles could pass the walls with the no slip condition. And this profile can be influenced by factors such as wall roughness, the channel geometry, and the characteristics of the flow.

Moreover, in microfluidic systems, multiphase flows are the norm in which two or more fluids reside within the same microchannel in two phase flow applications. They may be made of one liquid in one phase and another immiscible liquid or gas in the other. Such systems behavior is controlled by interfacial tension between the phases, which can make major difference in flow patterns, mixing and transport phenomena.

8. Conclusion

For devices that require controlled movement of fluid, fundamental understanding of microfluidic flow is necessary for the design and optimization of these. The knowledge of fluid dynamics at the micro scale allows one to enhance mixing for chemical reactions, and develop more efficient and effective microfluidic systems to improve heat dissipation for electronic cooling, as well as precision in biological assay. Studying the continuum of future technologies in medical diagnostic, drug delivery system or environmental monitoring requires the understanding of micro fluidic flow and its behaviour. To summarize, the modeling of fluid flow in microchannels with variable viscosity is a complicated however essential part of designing and improving microfluidic methods. The inclusion of viscosity dependence upon temperature, pressure, and fluid composition allows an engineer to accurately predict fluid behavior more so than with approach (a). The optimization is then translated into ideal shapes functionally effective in achieving the desired shape of either the actuation with the change of pressure in the case of pressure actuators, and the distribution of the flow in the case of flow actuators. Including variable viscosity into the mathematical models of microfluidic systems means that such systems will operate efficiently and reliably in the real world, where the fluids are seldom uniform in their properties.

References:

[1] Sung, Y.-T.; Chang, K.-E.; Liu, T.-C. The effects of integrating mobile devices with teaching and learning on students' learning performance: A meta-analysis and research synthesis. Comput. Educ. 2016, 94, 252–275.

[2] Chauhan, S. A meta-analysis of the impact of technology on learning effectiveness of elementary students. Comput. Educ. 2017, 105, 14–30.

[3] Felisoni, D.D.; Godoi, A.S. Cell phone usage and academic performance: An experiment. Comput. Educ. 2018, 117, 175–187.

[4] Safarie, M.; Tarlani-aliabadi, H. Who Practices What? A Correlational Study of Personality Type and Teaching Reflection. Procedia-Soc. Behav. Sci. 2014, 98, 1606–1640.

[5] Soisangwarn, A.; Wongwanich, S. Promoting the reflective teacher through peer coaching to improve teaching skills. Procedia-Soc. Behav. Sci. 2014, 116, 2504–2511.

[6] Kirk, D.E. Optimal Control Theory: An Introduction; Dover Publications: New York, NY, USA, 2012.

[7] Lenhart, S.M.; Workman, J.T. Optimal Control Applied to Biological Models; CRC Press: Boca Raton, FL, USA, 2007; Volume 15.

[8] Wang, X. Solving Optimal Control Problems with MATLAB: Indirect Methods; ISE Dept., NCSU: Raleigh, NC, USA, 2009.

[9] Daza, V.; Makriyannis, N.; Riera, C.R. MOOC attack: Closing the gap between pre-university and university mathematics. Open Learn. J. Open Distance E-Learn. 2013, 28, 227–238.

[10] Taranto, E.; Arzarello, F. Math MOOC UniTo: An Italian project on MOOCs for mathematics teacher education, and the development of a new theoretical framework. ZDM—Int. J. Math. Educ. 2019, 52, 843–858.

[11] Taranto, E. MOOC’s Zone theory: Creating a MOOC Environment for Professional Learning in Mathematics Teaching Education. Ph.D. Thesis, Turin University, Turin, Italy, 2018.

[12] Alves, I.F. The transnational phenomenon of individual planning in response to pupil diversity: A paradox in educational reform. In Critical Analyses of Educational Reforms in an Era of Transnational Governance; Springer: Cham, Switzerland, 2018; pp. 151–168.

[13] Demo, H. Didattica Aperta e Inclusione; Erickson: Trento, Italy, 2016.

[14] Norwich, B. Dilemmas of Difference, Inclusion and Disability: International Perspectives and Future Directions; Routledge: London, UK, 2008; Volume 1.

[15] Su, C.-H.; Cheng, C.-H. A mobile gamification learning system for improving the learning motivation and achievements: A mobile gamification learning system. J. Comput. Assist. Learn. 2015, 31, 268–286.

[16] Kapp, K.M. The Gamification of Learning and Instruction: Game-Based Methods and Strategies for Training and Education; Pfeiffer: San Francisco, CA, USA, 2012; ISBN 978-1-118-09634/5.

[17] Parra-González, M.E.; Segura-Robles, A. Producción científica sobre gamificación en educación: Un análisis cienciométrico. Rev. Educ. 2019, 5, 113–131.

[18] Lizarzaburu, A.; Mora, A.; Sánchez, A. Validez Interna y Externa de la Investigación Cualitativa. Rev. Contrib. A Las Cienc. Soc. 2015, 12, 1–6.

[19] Bardin, L. Análisis de Contenido; Akal: Madrid, Spain, 1996; pp. 20–35.

[20] Rodríguez, G.; Gómez, M.A. Análisis de Contenido y Textual de Datos Cualitativos. In Principios, Métodos y Técnicas Esenciales para la Investigación Educativa; Martín, S.N., Ed.; Dykinson: Madrid, Spain, 2010; pp. 447–469.

[21] Hsu, Y.C.; Ho, H.N.J.; Tsai, C.C.; Hwang, G.J.; Chu, H.C.; Wang, C.Y.; Chen, N.S. Research trends in technology-based learning from 2000 to 2009: A content analysis of publications in selected journals. J. Educ. Technol. Soc. 2012, 15, 354–370.

[22] Lavrakas, P.J. Encyclopedia of Survey Research Methods; SAGE: Thousand Oaks, CA, USA, 2008.

[23] Stanzione, D.; Barth, B.; Gaffney, N.; Gaither, K.; Hempel, C.; Minyard, T.; Mehringer, S.; Wernert, E.; Tufo, H.; Panda, D.; et al. Stampede 2. In Proceedings of the Practice and Experience in Advanced Research Computing 2017 on Sustainability, Success and Impact, New York, NY, USA, 9 July 2017.

[24] Stewart, C.A.; Hancock, D.Y.; Vaughn, M.; Fischer, J.; Cockerill, T.; Liming, L.; Merchant, N.; Miller, T.; Love, J.M.; Stanzione, D.C.; et al. Jetstream: Performance, early experiences, and early results. In Proceedings of the XSEDE16 Conference on Diversity, Big Data, and Science at Scale, Miami, FL, USA, 17–21 July 2016.

[25] Hancock, D.Y.; Stewart, C.A.; Vaughn, M.; Fischer, J.; Lowe, J.M.; Turner, G.; Swetnam, T.L.; Chafin, T.K.; Afgan, E.; Pierce, M.E.; et al. Jetstream—Early operations performance, adoption, and impacts. Concurr. Comput. Pract. Exp. 2019, 31, e4683.